

Math 10250 Activity 9: Compound Interest and the Number e (Sec. 2.2)

Last time: Let $A(t)$ be the balance at time t (years) of a bank account earning interest at an annual rate r (in decimals) compounded n times a year. Then we have:

$$A(t) = P \left(1 + \frac{r}{n}\right)^{tn}$$

where P is the principal i.e. $A(0) = P$.

Example 1 The balance $M(t)$ of a retirement account with interest compounded daily is given by the formula $M(t) = 30000(1.00022)^{365t}$. What is the principal and the annual interest rate?

(Ans: $P = \$30000$; $r = 8\%$)

Next, we want to consider the balance of an account where interest is compounded continuously i.e. we are earning interest every instant the money is with the bank. (Good deal?)

► The number e

In the general formula above, if $P = 1$, $r = 1$ and $t = 1$ then $A(1) = \left(1 + \frac{1}{n}\right)^n$.

Letting n go to ∞ we obtain that:

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = ? \quad \leftarrow \text{balance at end of 1 yr. of an investment of \$1 at an annual interest rate of 100\% compounded continuously}$$

Example 2 Estimate e by completing the table:

n	1	2	10	100	1000
$\left(1 + \frac{1}{n}\right)^n$					

Continuously compounded interest

Compute the limit:

$$\lim_{n \rightarrow \infty} \left(1 + \frac{r}{n}\right)^n = \quad = \quad =$$

\uparrow letting $m = n/r$, so that $n = mr$
 \uparrow by definition of e

Setting: As above except now $n \rightarrow \infty$

The amount after t years with **continuously compounded interest** is:

$$A(t) = \lim_{n \rightarrow \infty} P \left(1 + \frac{r}{n}\right)^{tn} = P \cdot \lim_{n \rightarrow \infty} \left[\quad \right]^t =$$

General formula:

$$A(t) = Pe^{rt}$$

\uparrow amount in account at end of t years \nwarrow initial amount invested (principal)

\leftarrow (rate)(time in years)

Example 3 If you open an account paying 9% interest, compounded continuously, then how much should you deposit to insure that there will be \$60,000 in 15 years?

Ans. $60,000e^{-1.35}$

Example 4 $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{2n}\right)^{3n} \stackrel{?}{=} \quad$

Ans. $e^{3/2}$

Example 5 Suppose you put \$5000 in an account paying 4% annual interest, and you leave it there without adding or withdrawing anything. How much will you have at the end of 3 years if the interest is compounded:

(a) 6 times a year?

Ans. \$5,635.24

(b) 24 times a year?

Ans. \$5,636.92

(c) continuously?

Ans. \$5,637.48

Remark: What could you conclude from the answers obtained in Example 5?

► The natural exponential function

Recall: The exponential function is $f(x) = b^x$, where b is a positive constant. The most **popular** b is e .

Definition: The **natural exponential function** is $f(x) = e^x$.

Example 6 Graph the natural exponential function and its inverse. Write down all intercepts and asymptotes of the natural exponential function.