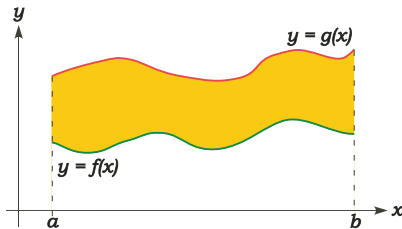


**Math 10250 Activity 36: More on Areas and Estimating Definite Integrals**  
(Section 5.6 continue & 5.7)

**GOAL:** To compute definite integrals and areas between the graphs of two functions. Introduce more numerical methods for integration.

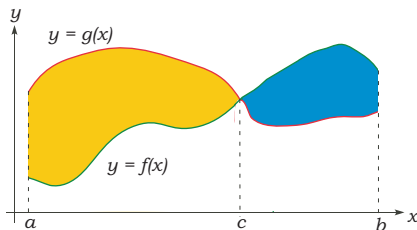
► **The area between two curves**

Consider the following region:



Area between $f$ and $g = \int_a^b \quad \quad \quad$ , if $f(x) \leq g(x)$ .
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Now consider:



Area between  $f$  and  $g$   
 $= \int_a^c [g(x) - f(x)] dx + \int_c^b [f(x) - g(x)] dx.$

**Example 1** Find the intersection points of  $f(x) = 1 - x^2$  and  $g(x) = x + 1$ . Then find the area between the graphs over the interval  $-1 \leq x \leq 1$ .

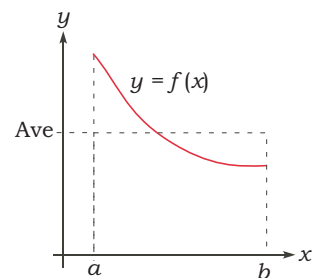
► **Average values of continuous quantities**

**Q1:** What is the average value of 3, 5 and 7?      **Answer:**

\_\_\_\_\_

**Q2:** What is the average value of  $f(x)$  on  $[a, b]$ ?

**A2:** Average value of  $f$  over  $[a, b] = \int_a^b$  \_\_\_\_\_

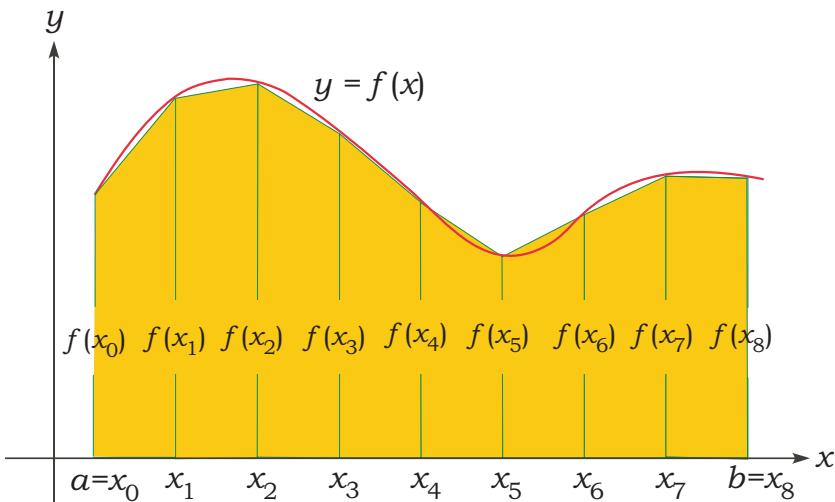


**Example 2** Find the average value of  $f(x) = x^5$  over the interval  $[-1, 2]$ .

**Example 3** Estimate  $\int_0^1 e^{\sqrt{x}} dx$   
 (a) using the mid-point with  $n = 4$

(b) using trapezoidal rule with  $n = 4$ .

► **The trapezoidal rule:** To estimate  $\int_a^b f(x) dx$ , we can use trapezoids instead of rectangles. Recall that the area of a trapezoid =  $\frac{1}{2}$ (sum of the two parallel sides)·(height). Apply this method for the function below.



$$\int_a^b f(x) dx \approx [f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)] \cdot \frac{\Delta x}{2} \leftarrow \text{trapezoidal rule}$$