Name _____ Date _____

**Math 10260 Activity 15: The Second Derivative Test (Section 7.3)**

**Goal:** Understand the **second derivative test** for a function $f(x, y)$.

**Q1:** What is the **second derivative test** for a function $f(x)$ of one variable?

**A1:**

Compare this now with:

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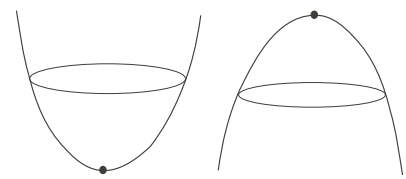
▶ **Second derivative test:** it Let $(x_0, y_0)$ be a critical point of a function $f(x, y)$, that is:

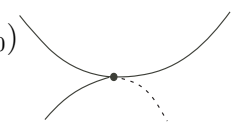
$$\frac{\partial f}{\partial x}(x_0, y_0) = 0 \text{ and } \frac{\partial f}{\partial y}(x_0, y_0) = 0,$$

and compute the "**determinant**" function: $D(x, y) = \left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$. Then:

- $D(x_0, y_0) > 0$ *and* $\frac{\partial^2 f}{\partial x^2}(x_0, y_0) > 0 \Rightarrow f(x, y)$ *has local minimum at* $(x_0, y_0)$
- $D(x_0, y_0) > 0$ *and* $\frac{\partial^2 f}{\partial x^2}(x_0, y_0) < 0 \Rightarrow f(x, y)$ *has local maximum at* $(x_0, y_0)$



- $D(x_0, y_0) < 0 \Rightarrow f(x, y)$ *has neither a local maximum nor a local minimum at* $(x_0, y_0)$ *(***saddle point***)*



- $D(x_0, y_0) = 0 \Rightarrow$ *No conclusion about extrema can be drawn .*

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**Example 1** For each of the following functions find the critical point and determine whether there is a local minimum, a local maximum, or neither at the point.

(a) $f(x, y) = x^2 + xy + y^2$

(b) $f(x, y) = -3x^2 + 3xy - 2y^2$

(c) $f(x, y) = -x^2 + 10xy - 2y^2$

(d) $f(x, y) = x^4 + y^4$

**Example 2** Determine whether $f(x, y) = x^3 - 6xy + 3y^2 - 9x + 7$ has local maximum or minimum.

▶ **GLOBAL Maxima and Minima for Quadratic Functions:** If a quadratic function has a local minimum (maximum) then it also has a global minimum (maximum) there.

**Example 3** Determine whether $f(x, y) = 4x^2 + 3y^2 - 4x - 6y + 1$ has gobal maximum or minimum.

(Answer: minimum at $x = 1/2, y = 1$)

**Example 4** A toy store carries two models of a certain board game−− the standard and deluxe−−. It has determined that if it charges $x$ dollars for the standard and $y$ dollars for deluxe, then it will sell $(45 - 3x + 2y)$ standard and $(6 + x - y)$ deluxe games per week.

**(a)** Write down a formula for the weekly revenue as a function of $x$ and $y$.

**(b)** Find $x$ and $y$ that maximize the revenue. How do you know that it gives a maximum and not a minimum or saddle point?

(Answer: maximum revenue at $x = 36, y = 57$)

**Example 5** Let $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2$. Find the critical points and determine whether there are local minima, a local maxima, or neither at the points.

(Answer: local minimum at $(2, 0)$,
Saddle points at $(1, 1)$ and $(1, -1)$,
and local maximum at $(0, 0)$)