

UNIVERSITY OF NOTRE DAME  
Department of Civil Engineering  
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CE 30125  
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**Homework Set #1**

**Background:** Taylor series provide a very useful and powerful tool in order to not only derive approximations in discrete space, but also to provide valuable information about the errors that we make by using a discrete approximation. There are two key things that we want to observe: (1) errors get larger as we apply larger discrete distances or move away from the expansion point; (2) errors have associated convergence rates - often dominated by the leading order truncated term. This information will be extremely useful in obtaining estimates of how large the numerical error is for our solution and will also provide guidance as to how build our discrete "grids" or networks of nodes at which the numerical solutions are computed.

**Problem 1**

Consider the function

$$f(x) = -0.78x^4 - 0.25x^3 - 0.45x^2 - 0.25x + 1.2$$

- a) Develop 2, 4, and 6 term Taylor series expansions  $g_{TS}(x)$  about  $x = 0$ .
- b) Plot the exact function,  $f(x)$ , as well as each of the three Taylor series expansions over the interval  $[0,1]$  using a plotting increment equal to at least 0.05.
- c) Plot the actual error,  $E(x) = f(x) - g_{TS}(x)$  and the estimated error,  $e(x)$ , which can be estimated by the leading order truncated term for each of the three Taylor series expansions. Plot these error measures against the distance about the expansion point,  $h$ , on log-log axes.
- d) How do the errors generally behave when the distance from the expansion point becomes larger?
- e) What is the slope on the log-log error plot for each of the expansions. How does this relate to the power of the distance from the expansion point for the leading order truncation term?
- f) Why is the 6 term expansion exact (i.e. both the estimated error and the actual error terms are equal to zero)?

**Background:** Many computational codes solving problems in mechanics, optimization, and/or forecasting will result in very large systems of algebraic equations that need to be solved. *Direct methods* are available that rely on systematically re-arranging the equations into much more convenient forms that allow for straight forward solutions. The cost of the solution is a vital factor and for example for Gauss elimination it cost  $O(N^3)$  operations where  $N$  is the size of the fully populated matrix. Accuracy is also a very important consideration. In the case of an *ill-conditioned* matrix, roundoff error may lead to significant problems. A *diagonally dominant* matrix will not experience ill-conditioning problems. So we should always check and re-arrange the system of equations to be diagonally dominant. In case this is not possible we can apply partial or full pivoting while solving the system of equations.

**Problem 2**

Solve the following system of linear equations *by hand* using Gauss elimination:

$$3x + 12y + 3z - 29u = 129$$

$$4x + 2y + 23z + 8u = 3$$

$$25x + 3y + 5z - 2u = 2$$

$$3x + 17y + 7z - 5u = 5$$

- a) Check for diagonal dominance and re-arrange the system if necessary
- b) Perform the lower triangularization. Keep track of your operation count.
- c) Perform the back-substitution to solve the system. Keep track of the number of operations.

### **Problem 3**

Consider the system of equations:

$$0.00025x + 3.00200y = 5.00215$$

$$1.00230x + 1.04511y = 1.01200$$

- a) Solve the system of linear equations **by hand** using Gauss elimination using 3 and 7 significant figures
- b) Solve the system of linear equations **by hand** using Gauss elimination with partial pivoting using 3 significant figures

*Note: Your calculator will retain about 10-11 significant digits of accuracy. Therefore you will have to round to the appropriate accuracy and actually re-enter the numbers in your calculator after you have computed them. Also use floating point representation of the numbers*

*([http://en.wikipedia.org/wiki/Floating\\_point](http://en.wikipedia.org/wiki/Floating_point))*

*Thus your 3 significant digit representation of 0.00025 is 2.50 E-04 and of 5.00215 is 5.00 E+00*